A New Method for Accelerating Impossible Differential Cryptanalysis and its Application on LBlock

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ABSTRACT

Impossible differential cryptanalysis, an extension of the differential cryptanalysis, is one of the most efficient attacks against block ciphers. This cryptanalysis method has been applied to most of the block ciphers, and has shown significant results. Using structures, key schedule considerations, early abort, and pre-computation are some common methods to reduce complexities of this attack. In this paper, we present a new method for decreasing the time complexity of impossible differential cryptanalysis through breaking down the target key space into subspaces, and extending the results on subspaces to the main target key space. The main advantage of this method is that there is no need to consider the effects of changes in the values of independent key bits on each other. Using the 14-round impossible differential characteristic observed by Boura et al. at ASIACRYPT 2014, we implement this method on 23-round LBlock and demonstrate that it can reduce the time complexity of the previous attacks to $2^{71.8}$ 23-round encryptions using $2^{59}$ chosen plaintexts and $2^{73}$ blocks of memory.

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1 Introduction

1.1 Motivation, Contribution and Organization

Cryptanalysis methods that rely on eliminating the round’s function key’s effect by working on the differences, form a considerable part of attacks against block ciphers. Differential cryptanalysis [1] proposed by Biham et al. is the first attack of this category. Impossible differential, higher order differential, truncated differential, and boomerang cryptanalyses are other attacks of this type.

Impossible differential cryptanalysis was introduced independently by Knudsen on DEAL block cipher [7] and Biham et al. on Skipjack block cipher [2]. It is one of the conventional cryptanalyses methods for block ciphers showing remarkable results. Distinguisher of impossible differential attack, impossible differential characteristic, is $n$-round of target algorithm with specific input and output differences holding with probability zero, implying that a pair with such input difference cannot lead to the specified output difference after $n$ rounds. After extending the characteristic by adding some rounds to the plaintext and ciphertext sides of the distinguisher, we try to find the key values leading chosen pairs of plaintexts to the attack’s
distinguisher. Since these key values lead to the distinguisher holding with probability zero; they cannot be the correct key and should be eliminated from the target key space.

LBlock is a lightweight block cipher of generalized Feistel type represented in ACNS2011 [3]. The algorithm consists of 32 rounds with 64-bit block length, and 80-bit key length. In [3], the designers evaluated LBlock’s security against different attacks including impossible differential cryptanalysis. Using a 14-round impossible differential characteristic they presented the attack on 20 rounds of the algorithm. Using another 14-round characteristic, impossible differential cryptanalysis on 21 rounds of the algorithm is presented in [4]. This improvement was achieved by applying mentioned methods, such as key schedule consideration. The authors in [5] represented another impossible differential cryptanalysis of LBlock. They have separated the plaintext and ciphertext sides added to the distinguisher and searched each part independently. Using this method, they could reduce the complexity on 21 rounds of algorithm. Also, they have extended the attack to 22 rounds. Our new method can be considered as an extension of their work. By investigating the key schedule of the cipher in [9], the authors have extended impossible differential cryptanalysis to 23 rounds of the algorithm. In [10], the authors presented another impossible differential cryptanalysis on 23 rounds of the algorithm by using an approach that can help reducing the number of pairs used in the attack.

In conventional impossible differential attacks, the effects of changes in most of the target key bits are considered while the attackers are studying the possible values for some specific key bits, although lots of them are independent; and these considerations come to attacks with higher time complexities. The main idea of the new method is preventing this overload by separating the target key space into subspaces. We classify the target key bits into the groups and determine the values of key bits in each group independently. Then, we extend the achieved results to the whole target key space. This idea was first applied on 22-round LBlock in [21]; however, there was no efficient algorithm for extending the achieved results to the main target key space. In this paper, we generalize the method and propose an efficient algorithm for combining the results stored in distinct tables.

Using this improved method in parallel with previous ones, an improved attack on 23-round LBlock is presented in this paper. The results of this attack are compared with the previous works in Table 1. The outstanding advantages of this method are its generality that can be applied on different block ciphers (especially those with weak diffusion layers including lightweight algorithms) and also applicability in parallel with other techniques proposed for improving impossible differential cryptanalysis. The rest of this paper is organized as follows: In Section 2, we introduce the impossible differential cryptanalysis and describe the new method. Prevalent notations for LBlock and brief description of it are presented in Section 3. In Section 4, we clarify the new method by applying it on LBlock. Finally, we conclude the paper in Section 5.

### 2 Impossible Differential Cryptanalysis and the New Method

Consider the typical round function in block ciphers of Feistel type, consisted of add round key by XOR op-
One page of the document contains a detailed explanation of impossible differential cryptanalysis, a method used in cryptography to analyze the security of cryptographic algorithms. The text includes definitions, properties, and theorems related to the impossible differential attack, which involves finding pairs of plaintexts and ciphertexts that lead to a specific difference in the output of the cipher's S-boxes.

The page begins with a definition of cancellation in the context of Feistel ciphers, followed by a discussion of the terms used in the paper. The authors define various concepts such as the size of involved-key-bits, conditions of the impossible differential attack, and similar terms. They also discuss the impossibility of differential cryptanalysis, providing a lower bound for the time complexity of such an attack.

The page contains mathematical expressions and inequalities, such as $X \oplus Y$ for XOR operations and $S(X \oplus K)$ to represent S-box operations. The text explains how to determine the values of $K$ leading to a specified S-box output and how to test the possible key values on them to find incorrect key values.

Overall, the page provides a comprehensive overview of impossible differential cryptanalysis, including its conditions, properties, and theorems, making it a valuable resource for understanding this aspect of cryptography.

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**Figure 1.** Typical round function in algorithms of Feistel ciphers

**Figure 2.** Impossible differential cryptanalysis
tack, which gives $2^e$ values for the $L$-bit involved keys, contains at least $(L - e + \ln(2)) \times 2^e$ memory accesses.

In conventional impossible differential cryptanalysis, there are some common methods to reduce the attack’s complexity; such as using structures, key schedule considerations, early abort, and pre-computation. Using structures leads attackers to choose texts of specific form in which there are constant values in some positions of the texts’ block and the other positions can take all possible values. By this method, the chosen pairs from one structure have the desired differential form. Key schedule consideration is one of the most effective methods to reduce the complexity of attacks. The key schedule generates rounds’ subkeys from one seed, main key, usually in an iterative form; and therefore, it contains some redundancy. In ideal key schedules, this redundancy is not exploitable; but for most of the algorithms, the attackers can find some applicable redundancy to reduce size of the target key space. In early abort technique, attackers discard the pairs or key values that cannot comply with some conditions as soon as possible. Here, pre-computation means performing the repetitive computations that are independent from the achieved results of the attack’s steps and classifying them in tables before the attack starts. Then, attackers can refer to these tables and find the result of the intended computation when they need, instead of spending time on computing in the online phase of the attack.

2.2 The New Method

In impossible differential attack, the attackers are searching for the values of the information-key-bits satisfying the attack’s conditions to discard them from the target key space. Not necessarily all the information-key-bits have effect on all the conditions. However, in conventional impossible differential cryptanalysis methods, the effects of changes in the values of all the information-key-bits are considered while attackers are studying a specific cancellation. Here, we present a new method which is based on reducing computational complexity by refusing to study the effects of independent key bits’ changes on the attack’s conditions.

In our attack scenario, for each one of $l_1 + l_2$ cancellations, we determine the effective key bits and put them in distinct groups. Then, for each group, we form a table consisted of flags corresponding to all the possible values of the determined key bits. For each chosen pair, we find the key values satisfying each cancellation and set the achieved values’ flags in the corresponding table. The combinations of key values that their flags are set from the tables lead the chosen pair to the attack’s distinguisher and should be eliminated from the target key space.

According to property 1, we just need to know the input pair value of $(X_1, X_2)$ and the desired output difference of the F-function $F = S(X \oplus K)$ for each cancellation to determine the key values of the function; and just the changes in the key bits that affect the input pair value or the desired differential output value of the F-function are needed to be considered, not the changes in all the involved-key-bits. Therefore, there are two types of key bits in each group:

Type-1: key bits of the F-function on which we have cancellation,

Type-2: key bits which affect the input pair value or the desired differential output value of this function.

(Groups of key bits corresponding to cancellations on the first and last rounds do not contain key bits of type-2.)

For each group of key bits that contains key bits of type-2, we guess this type of key bits to determine the input pair value and the desired differential output value of the F-function. Then, we can determine the values of type-1 key bits in each group.

Impossible differential cryptanalysis by using the new method can be done within the following steps:

1. Choosing a proper $n$-round impossible differential characteristic,
2. Extending number of rounds to $n + r_1 + r_2$,
3. Determining the non-zero difference sub-blocks of the plaintext and ciphertext sides,
4. Specifying number of cancellations $l_1 + l_2$ and their locations,
5. Forming $l_1 + l_2$ key groups corresponding to $l_1 + l_2$ cancellations and determining each group’s key bits,
6. Forming $l_1 + l_2$ tables corresponding to $l_1 + l_2$ key groups consisted of flags corresponding to the possible values of the determined key bits for each group,
7. Choosing pairs with the specified difference in the plaintext and ciphertext sides, and repeating the two following steps for each pair,
8. Determining key bits values of each groups and setting their corresponding values’ flags in the corresponding table,
9. Eliminating the combinations of key bits values of tables that their flags are set from the target key space.

We clarify this method in Section 4 by applying it on 23-round LBlock. To the best of our knowledge, impossible differential attack by this method comes to the best results in terms of data and time complexities, considering the cryptanalysis methods in single-key.
model excluding biclique attack.

3 Brief Description of LBlock

The used notation in this paper is summarized here:

- $A$: a bit string
- $A|B$: concatenation of strings $A$ and $B$
- $A <<< B$: left rotation of $A$ by $j$ bits
- $K_i^r$: the $i^{th}$ nibble of the $r^{th}$ round key
- $R_i^{r-1}$: the right half of the $r^{th}$ round’s input
- $L_i^{r-1}$: the left half of the $r^{th}$ round’s input
- $R_i^r$: the $i^{th}$ nibble of $R_i^{r-1}$
- $L_i^r$: the $i^{th}$ nibble of $L_i^{r-1}$
- $\Delta R_i^{r-1}$: the difference of two $R_i^{r-1}$
- $\Delta L_i^{r-1}$: the difference of two $L_i^{r-1}$
- $\Delta R_i^r$: the difference of two $R_i^r$
- $\Delta L_i^r$: the difference of two $L_i^r$
- $S_i^r$: output of the $i^{th}$ S-Box in the $r^{th}$ round
- $\Delta S_i^r$: the difference of two $S_i^r$

(\text{Note that the nibbles are indexed beginning with zero subscript and ending with seven, and the zero-indexed nibble is the rightmost one.)}

LBlock is a lightweight block cipher with 64-bit block length and 80-bit key length [3]. Its structure is a generalization of a Feistel structure with 32 rounds. It can be implemented efficiently both in hardware and software. The iterative round function $F$ consists of three basic functions: (1) add round key by XOR, (2) confusion function by S-Boxes, and (3) diffusion function by permutation and rotation. LBlock’s algorithm uses 4-bit S-Boxes $S_i$, $i = 0, 1, \ldots, 9$; where $S_i$, $i = 0, 1, \ldots, 7$ are used in the round functions and S-Boxes $S_9$ and $S_{10}$ are used in the key schedule. Its round function is depicted in Figure 3.

![Figure 3. Round function of LBlock.]

4 Improved Impossible Differential Cryptanalysis on LBlock

S-Boxes used in an algorithm are so effective in the security against cryptanalyses. After studying differential distribution tables of the S-Boxes of LBlock, $S_i$, $i = 0, 1, \ldots, 7$ we found a property common among them:

**Property 2** For the SBoxes used in the LBlock’s algorithm, the probability of generating a specific output difference from an input difference is $2^{-(i-4)}$; and for each possible input pair and output difference, there are on average $2^{14}$ key value.

4.1 Impossible Differential characteristic

We use the 14-round impossible differential characteristic as our attack’s distinguisher [10]. This characteristic is as follows (see Figure 5 in the Appendix):

$$(0000, 0000, 0000, a000) \rightarrow (0000, 0600, 0000, 0000)$$

Where $a, b \in \{0, 1\}^4 \setminus \{0\}^4$ are two non-zero nibbles. We add five rounds to the plaintext side and four rounds to the ciphertext side of the characteristic and propose our attack on 23-round LBlock (Figure 4).

4.2 Separating Target Key Space into Subspaces

There are eighteen cancellations during the attack; and therefore, we separate target key bits into eighteen groups. The groups $g_i, i = 1, 2, 3, \ldots, 18$ and their corresponding conditions are demonstrated in Table 2. We find key values of each group, committing the corresponding cancellation, and discard the combinations of these keys values from the target key space.

4.3 Pre-computation

For $S_i, i = 0, 1, \ldots, 7$ we form the tables $A_i, i = 0, 1, \ldots, 7$ indexed by $(x_1, x_2, \Delta y)$. For all possible values of $(x_1, x_2, \Delta y)$, we find key values which result specific values for $\Delta y = [S_i(x_1 + k) \oplus S_i(x_2 + k)], i = 0, 1, \ldots, 7$; and put them in the corresponding rows of $A_i, i = 0, 1, \ldots, 7$. Also, for 18 groups of key bits, we form tables $U_i, i = 1, 2, 3, \ldots, 18$ consisted of flags corresponding to the possible values for the key bits in each group.

4.4 Key Recovery

The key recovery procedure is organized as the following steps:

**Step 1** (Choosing plaintexts). Take $2^n$ structures of the form $(b_1|a_1|b_2|a_2)|\cdots|b_9|b_3|a_3)\cdots\langle b_{10}|b_{11}|b_{12}|a_4)\rangle$ where $a_i, i = 1, 2, 3, 4$, are 4-bit fixed constants, and each $b_i, i = 1, 2, 3, \ldots, 12$ takes all $2^4$ possible values for a nibble. Almost all the chosen pairs of each structure have differential form of $((\ast|\ast|\ast|0)|\langle (\ast|\ast|\ast|\ast)|\langle (\ast|\ast|\ast|\ast)|\rangle$ where “$\ast$” presents a non-zero 4-bit value. Each struc-
we can check if the pair can satisfy the cancellations or not. According to the Property 2, as the differential input to check if an input pair with S-boxes of the algorithm cannot lead to their ciphertext difference comply with (\(000)\) or \(000\) of the ciphertext side). Encrypt the input pair values and the desired output difference of the corresponding F-functions. Set the flags of the corresponding group of key bits.

For each one of the \(2^{n+42}\) survived pairs repeat the following steps.

Step 4 (Cancellations 1, 2, 3, 4, 12, 13, 14). The key values of groups 1, 2, 3, 4, 12, 13 and 14 can be determined by referring to the tables \(A_1, A_2, A_3, A_4, A_5, A_6\), respectively (since we know the input pair values and the desired output difference of the corresponding F-functions). Set the flags of the obtained key values for the cancellations 1, 2, 3, 4, 12, 13 and 14 in tables \(U_1, U_2, U_3, U_4, U_{12}, U_{13}, U_{14}\), respectively. This step contains \((7 + (7 \times 2^{1.4}))\) memory accesses to find the key values and setting the flags.

Step 5 (Cancellations 5, 10, 11). In step 3, we have filtered the differential pairs, using the differential distribution tables of the S-Boxes corresponding to cancellations 1-18, except 5, 10, and 11. This filtration was possible since the differential input and output of those cancellations were known having the values of the plaintexts and ciphertexts (Table 3). However, since either the differential input or output of S-Boxes corresponding to cancellations 5, 10, and 11 is \(\Delta L_3\). (Table 4) which is dependent to the subkey \(K_3\), we put off verification for these cancellations until we

### Table 2. Conditions of the Cancellations and Corresponding Groups of Key Bits.

<table>
<thead>
<tr>
<th>NO.</th>
<th>Condition</th>
<th>Corresponding group of key bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(S_1(L_1^0 \oplus K_1^1) \oplus S_1(L_1^0 \oplus K_1^1) = \Delta R_0^6)</td>
<td>(g_1 = {K_1^1})</td>
</tr>
<tr>
<td>2</td>
<td>(S_2(L_2^3 \oplus K_2^3) \oplus S_2(L_2^3 \oplus K_2^3) = \Delta R_2^4)</td>
<td>(g_2 = {K_2^3})</td>
</tr>
<tr>
<td>3</td>
<td>(S_2(L_2^4 \oplus K_2^4) \oplus S_2(L_2^4 \oplus K_2^4) = \Delta R_2^4)</td>
<td>(g_3 = {K_2^4})</td>
</tr>
<tr>
<td>4</td>
<td>(S_5(L_5^5 \oplus K_5^5) \oplus S_5(L_5^5 \oplus K_5^5) = \Delta R_5^2)</td>
<td>(g_4 = {K_5^5})</td>
</tr>
<tr>
<td>5</td>
<td>(S_5(L_5^6 \oplus K_5^6) \oplus S_5(L_5^6 \oplus K_5^6) = \Delta R_5^2)</td>
<td>(g_5 = {K_5^6})</td>
</tr>
<tr>
<td>6</td>
<td>(S_7(L_7^7 \oplus K_7^7) \oplus S_7(L_7^7 \oplus K_7^7) = \Delta R_7^3)</td>
<td>(g_6 = {K_7^7})</td>
</tr>
<tr>
<td>7</td>
<td>(S_6(L_6^4 \oplus K_6^4) \oplus S_6(L_6^4 \oplus K_6^4) = \Delta R_6^2)</td>
<td>(g_7 = {K_6^4})</td>
</tr>
<tr>
<td>8</td>
<td>(S_7(L_7^5 \oplus K_7^5) \oplus S_7(L_7^5 \oplus K_7^5) = \Delta R_7^3)</td>
<td>(g_8 = {K_7^5})</td>
</tr>
<tr>
<td>9</td>
<td>(S_7(L_7^6 \oplus K_7^6) \oplus S_7(L_7^6 \oplus K_7^6) = \Delta R_7^3)</td>
<td>(g_9 = {K_7^6})</td>
</tr>
<tr>
<td>10</td>
<td>(S_7(L_7^7 \oplus K_7^7) \oplus S_7(L_7^7 \oplus K_7^7) = \Delta R_7^3)</td>
<td>(g_{10} = {K_7^7})</td>
</tr>
<tr>
<td>11</td>
<td>(S_5(L_5^6 \oplus K_5^6) \oplus S_5(L_5^6 \oplus K_5^6) = \Delta R_5^2)</td>
<td>(g_{11} = {K_5^6, K_5^7, K_5^8})</td>
</tr>
<tr>
<td>12</td>
<td>(S_6(L_6^4 \oplus K_6^4) \oplus S_6(L_6^4 \oplus K_6^4) = \Delta L_2^3)</td>
<td>(g_{12} = {K_6^3})</td>
</tr>
<tr>
<td>13</td>
<td>(S_6(L_6^4 \oplus K_6^4) \oplus S_6(L_6^4 \oplus K_6^4) = \Delta L_2^3)</td>
<td>(g_{13} = {K_6^3})</td>
</tr>
<tr>
<td>14</td>
<td>(S_6(L_6^4 \oplus K_6^4) \oplus S_6(L_6^4 \oplus K_6^4) = \Delta L_2^3)</td>
<td>(g_{14} = {K_6^3})</td>
</tr>
<tr>
<td>15</td>
<td>(S_6(L_6^4 \oplus K_6^4) \oplus S_6(L_6^4 \oplus K_6^4) = \Delta L_2^3)</td>
<td>(g_{15} = {K_6^3, K_6^7})</td>
</tr>
<tr>
<td>16</td>
<td>(S_6(L_6^4 \oplus K_6^4) \oplus S_6(L_6^4 \oplus K_6^4) = \Delta L_2^3)</td>
<td>(g_{16} = {K_6^3, K_6^7})</td>
</tr>
<tr>
<td>17</td>
<td>(S_6(L_6^4 \oplus K_6^4) \oplus S_6(L_6^4 \oplus K_6^4) = \Delta L_2^3)</td>
<td>(g_{17} = {K_6^3, K_6^7})</td>
</tr>
<tr>
<td>18</td>
<td>(S_6(L_6^4 \oplus K_6^4) \oplus S_6(L_6^4 \oplus K_6^4) = \Delta L_2^3)</td>
<td>(g_{18} = {K_6^3, K_6^7})</td>
</tr>
</tbody>
</table>
guess the values for $K_2$ in this step. For $2^4$ possible values of $K_2$, calculate $2^4$ values of $(L_{0}, L'_{0})$. Check the differential distribution table of S-Boxes $S_5$, $S_7$ and $S_3$ for cancellations 5, 10 and 11, respectively; and discard the values of $K_2$ that their corresponding values in the differential distribution tables are zero. According to property 2, the probability of surviving from these filtrations is $(2^{-1.4})^3 = 2^{-4.2}$; and $2^4 \times 2^{-4.2} = 2^{-0.2}$ values of $K_2$ pass this step. This step contains $(22)^4$ F-function and $2^4 + 2^4 + 2^4$ memory accesses.

Step 6 (Cancellation 5). For $2^{-0.2}$ possible values of $K_6^4$, calculate $2^{-0.2}$ values of $(L_{5}, L'_{5})$. Refer to the $(L_{5}, L'_{5}, \Delta R_{5})$ row of table $A_5$ to find the values of $K_7$. Set the flags of the achieved values for $(K_3^4|K_2^4)$ in table $U_5$. This step contains $2 \times 2^4$ F-function and $(2^{-0.2} + 2^{-0.2})$ memory accesses.

Step 7 (Cancellation 6). For $2^4$ possible values of $K_6^4$, calculate $2^4$ values of $(L_{4}, L'_{4})$. Refer to the $(L_{4}, L'_{4}, \Delta R_{4})$ row of table $A_7$ to find the values of $K_7$. Set the flags of the achieved values for $(K_4^4|K_2^4)$ in table $U_7$. This step contains $2 \times 2^4$ F-function and $(2^4 + 2^4)$ memory accesses.

Step 8 (Cancellation 7). For $2^4$ possible values of $K_1^4$, calculate $2^4$ values of $(L_{0}, L'_{0})$. Refer to the $(L_{0}, L'_{0}, \Delta R_{0})$ row of table $A_0$ to find the values of $K_2$. Set the flags of the achieved values for $(K_1^4|K_2^4)$ in table $U_7$. This step contains $2 \times 2^4$ F-function and $(2^4 + 2^4)$ memory accesses.

Step 9 (Cancellation 8). For $2^{1.4}$ possible values of $K_2^5$ from table $U_3$, and $2^2$ possible values of $K_2^7$, calculate $2^{1.4} \times 2^2 = 2^{2.4}$ memory accesses.

Step 10 (Cancellation 9). For $2^8$ possible values of $K_1^5[K_2^5, K_2^7]$, calculate $2^8$ values of $(L_{5}, L'_{5})$. Refer to the $(L_{5}, L'_{5}, \Delta R_{5})$ row of table $A_3$ to find the values of $K_3$. Set the flags of the achieved values for $(K_1^5|K_2^5|K_3^7)$ in table $U_3$. This step contains $2 \times 2^8$ F-function and $(2^8 + 2^8)$ memory accesses.

Step 11 (Cancellation 10). For $2^{-1.4}$ possible values of $K_7^4$ from step 5, $2^{-1.4}$ values of $K_1^4$ in table $U_4$, and $2^4$ possible values of $K_2^5[K_2^7]$, calculate $2^4$ values of $(L_{3}, L'_{3})$. Refer to the $(L_{3}, L'_{3}, \Delta R_{3})$ row of table $A_7$ to find the values of $K_7$. Set the flags of the achieved values for $(K_3^7|K_2^5|K_3^7)$ in table $U_7$. This step contains $2 \times 2^4$ F-function and $(2^4 + 2^4)$ memory accesses.
values of $K^4_3$ in table $U_3$, $2^{1.4}$ values of $K^4_1$ in table $U_1$, $2^{3.35}$ values of $K^4_3$ using table $U_1$, $2^{2.7}$ values of $K^3_0$ using table $U_4$, and $2^8$ possible values of $K^2_0 | K^2_1$, calculate $2^{18.25}$ values of $(L^4_1, L^4_1)$, for $2^{-0.2}$ possible values of $K^3_1$ from step 5, calculate $2^{-0.2}$ possible values of $\Delta R^3_2$. For $2^{18.25} \times 2^{-0.2} = 2^{18.05}$ possible values of $(L^4_3, L^4_3, \Delta R^4_2)$, refer to the $(L^4_3, L^4_3, \Delta R^4_2)^{th}$ row of table $A_3$ to find the values of $K^5_3$. Set the flags of the achieved values for $K^5_3 | K^5_1 | K^5_2 | K^5_3 | K^5_4 | K^5_5 | K^5_9 | K^5_2 | K^5_3$ in table $U_{11}$. This step contains $2 \times (2^{18.25} + 2^{-0.2})$ F-function and $(2^{18.05} + 2^{19.45})$ memory accesses.

**Step 13 (Cancellations 15).** For $2^4$ possible values of $K^2_0$, calculate $2^4$ values of $(L^2_1, L^2_1)$. Refer to the $(L^2_1, L^2_1, \Delta L^2_0)^{th}$ row of table $A_1$ to find the values of $K^2_2$. Set the flags of the achieved values for $(K^2_3 | K^2_4)$ in table $U_{15}$. This step contains $2 \times 2^4$ F-function and $(2^4 + 2^3 \times 4)$ memory accesses.

**Step 14 (Cancellation 16).** For $2^4$ possible values of $K^2_3$, calculate $2^4$ values of $(L^2_1, L^2_1)$. Refer to the $(L^2_1, L^2_1, \Delta L^2_0)^{th}$ row of table $A_1$ to find the values of $K^2_2$. Set the flags of the achieved values for $(K^2_3 | K^2_4)$ in table $U_{16}$. This step contains $2 \times 2^4$ F-function and $(2^4 + 2^3 \times 4)$ memory accesses.

**Step 15 (Cancellation 17).** For $2^8$ possible values of $K^3_3 | K^3_4$, calculate $2^8$ values of $(L^2_0, L^2_0)$. Refer to the $(L^2_0, L^2_0, \Delta L^2_0)^{th}$ row of table $A_3$ to find the values of $K^3_3$. Set the flags of the achieved values for $K^3_3 | K^3_4$ in table $U_{17}$. This step contains $2 \times 2^8$ F-function and $(2^8 + 2^4 \times 3)$ memory accesses.

**Step 16 (Cancellation 18).** For $2^{16}$ possible values of $K^3_3 | K^3_4 | K^3_5 | K^3_6 | K^3_7$, calculate $2^{16}$ values of $(L^2_0, L^2_0, L^2_0)$. Refer to the $(L^2_0, L^2_0, \Delta L^2_0)^{th}$ row of table $A_3$ to find the values of $K^3_3$. Set the flags of the achieved values for $K^3_3 | K^3_4 | K^3_5 | K^3_6 | K^3_7 | K^3_8$ in table $U_{17}$. This step contains $2 \times 2^{16}$ F-function and $(2^{16} + 2^{17} \times 4)$ memory accesses.

Now, we have the key values for 18 cancellations in tables; and the combinations of these key values are the incorrect keys that should be eliminated from the candidate key space.

**Step 17 (Combining the results).** In this step, we combine the achieved results stored in tables $U_i, i = 1, 2, \ldots, 17, 18$ to find the incorrect key values and eliminate them. Algorithm 1 shows an efficient method for this purpose. In this algorithm, we use the redundancy of the key schedule to reduce the number of times needed to access the tables. In this way, for a chosen combination of subkeys values from tables $U_i U_k, k = 1, 2, \ldots, 17$ that their corresponding flags are set, we just check the flags of the subkeys values in table $U_{i(k+1)}$ that they comply with the chosen combination according to the redundancy of the key.

![Figure 4. Round function of LBlock. White boxes represent 4-bit zero difference; black boxes present 4-bit non-zero differences; boxes containing question mark present 4-bit either zero or non-zero differences; and boxes containing black circle show the locations of the cancellations.](image-url)
Algorithm 1 Eliminating the combinations of the achieved results in tables $U_i$, $i = 1, \ldots, 18$ from target key space.

for $2^{(n+42)}$ pairs do
    for the subkey values in $g_1$ do
        if the flag is set in $U_1$ then
            for the the subkey values in $g_2 - g_1 g_2$ do
                if the flag is set in $U_2$ then
                    for the the subkey values in $g_3 - g_1 g_3 - g_2 g_3$ do
                        if the flag is set in $U_3$ then
                            :;
                            for the the subkey values in $g_8 - g_1 g_8 - g_2 g_8 - g_9 g_8 - \ldots - g_1 g_9 g_8$ do
                                if the flag is set in $U_{18}$ then
                                    eliminate the 73-bit key value from
                                    the target key space
                                end if
                            end for
                        end if
                    end for
                end if
            end for
        end if
    end for
end for

Table $U_{(k+1)}$ for a chosen combination from tables $U_1 \ldots U_k, k = 1, 2, \ldots, 17$, we minimize the time complexity of this step by using the redundancy of the key schedule. In other word, we do not access tables $U_i, i = 1, 2, 3, \ldots, 18$ to check the flags of the subkeys values that cannot be correct according to the key schedule; and reduce the time complexity of this step in this way. Considering the redundancy of the key schedule, there are $2^{73}$ candidates for the key value in tables $U_i, i = 1, \ldots, 18$, for being eliminated from target key space [10]. Among these $2^{73}$ values, we just pick those that their corresponding flags are set in tables $U_i, i = 1, \ldots, 18$. The probability of this condition being true for each key value is $2^{15(-2.6)+3(-4)} = 2^{(-51)}$, where $2^{(-2.6)}$ is the probability for each one of the 15 conditions 1, 2, \ldots, 18, excluding conditions 5, 10 and 11, and $2^{(-4)}$ is the probability for each one of the conditions 5, 10, and 11 holding true. So, the time complexity of this step is $2^{(n+42)} \times 2^{73} \times 2^{(-51)} \times 19 \approx 2^{(n+68.2)}$ memory accesses (18 memory accesses for tables $U_i, i = 1, \ldots, 18$, and one memory access for eliminating the incorrect key value).

4.5 Complexity

Since the attack contains $18 \times 4 = 72$ bits sieving, condition bits, the probability of being chosen for each key value after studying each pair is $2^{-72}$. So, the probability of not being chosen for each key value after studying each pair is $(1 - 2^{-72})$. The number of information-key-bits is $73$ [10], and after studying $m$ pairs, $N = 2^{73}(1 - 2^{-72})^m$ key values remain in the target key space. Putting $m = 2^{74}$, the number of remained key values is $N = 2^{67.2}$; and we find the correct key among the remained key by exhaustive search. From $m = 2^{n+63} = 2^{74}$, $n$ is $11$; and the data complexity of attack is $2^{n+48} = 2^{69}$ chosen plaintexts.

Since we eliminate the incorrect key values from the target key space, we need a memory of size $2^{73}$ to keep the whole possible values for the target key bits and eliminate the incorrect values among them. So, the memory complexity of the attack is $2^{73}$, and sizes of the other used memories are negligible.

The time complexities of the attack’s steps are indicated in Table 5. Dominant parts of time complexity refer to steps 3 and 17. Putting $n = 11$, the time complexity of attack is equivalent to $(2^{11+65} + 2^{11+68.2}) \times \left(\frac{1}{8} \times \frac{1}{23}\right) = 2^{71.8}$ 23-round encryption.

5 Conclusion

In this paper, we presented a new method for impossible differential cryptanalysis, based on separating target key space into subspaces. To this aim, we search the subspaces separately; then, we extend the combinations of the results to the whole target key space. We applied this method on 23 rounds of LBlock. The presented cryptanalysis requires $2^{69}$ chosen plaintexts and the time complexity is equivalent to $2^{71.8}$ encryptions, which is the best achieved results to the best of our knowledge. The achieved improvement is independent to those that are proposed so far; and studying their superposition property may lead to more efficient impossible differential attacks. Moreover, despite the fact that the proposed method does not necessarily lead to the most efficient impossible differential attack for all block cipher algorithms, its consideration is important while evaluating the security of an algorithm against this cryptanalysis method.
Table 5. Impossible differential cryptanalysis of 23-round LBlock

<table>
<thead>
<tr>
<th>Step No.</th>
<th>Target Group</th>
<th>Target Key Bits</th>
<th>No. of Survived Key Values per Pair</th>
<th>Time Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>None</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+44}$ 23-round encryption</td>
</tr>
<tr>
<td>3</td>
<td>None</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+65}$ memory accesses</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times (1 + 2^{14})$ memory accesses</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times (2^4 + 2^{16})$ memory accesses</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times (2^7 + 2^{14})$ memory accesses</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times (2^5 + 2^{16})$ memory accesses</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>$K_0$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times (2^5 + 2^{16})$ memory accesses</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>$K_0$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times (2^6 + 2^{17})$ memory accesses</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>$K_0$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times (2^6 + 2^{17})$ memory accesses</td>
</tr>
<tr>
<td>5</td>
<td>5, 10, 11</td>
<td>$K_1$</td>
<td>$2^{0.2}$</td>
<td>$2^{n+42} \times 2 \times 2^{11}$ p-function</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>$K_1$</td>
<td>$2^{0.2}$</td>
<td>$2^{n+42} \times 2 \times 2^{11}$ p-function</td>
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<tr>
<td></td>
<td>7</td>
<td>$K_2$</td>
<td>$2^4$</td>
<td>$2^{n+42} \times (2^4 + 2^{16})$ memory accesses</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>$K_2$</td>
<td>$2^4$</td>
<td>$2^{n+42} \times (2^4 + 2^{16})$ memory accesses</td>
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<tr>
<td></td>
<td>9</td>
<td>$K_2$</td>
<td>$2^4$</td>
<td>$2^{n+42} \times (2^6 + 2^{18})$ memory accesses</td>
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<tr>
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<td>10</td>
<td>$K_2$</td>
<td>$2^4$</td>
<td>$2^{n+42} \times (2^8 + 2^{16})$ memory accesses</td>
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<tr>
<td>11</td>
<td>10</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times 2 \times 2^{10}$ F-function</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times 2 \times 2^{10}$ F-function</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times 2 \times 2^{10}$ F-function</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times 2 \times 2^{10}$ F-function</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times 2 \times 2^{10}$ F-function</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times 2 \times 2^{10}$ F-function</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>$K_1$</td>
<td>$2^{14}$</td>
<td>$2^{n+42} \times 2 \times 2^{10}$ F-function</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>None</td>
<td>$2^{n+42} \times (2^{73} + 2^{51} + 2^{19})$</td>
<td>$2^{n+68}$ 23-round encryption</td>
</tr>
</tbody>
</table>

References


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Appendix

Figure 5. 14-round impossible differential characteristic. White boxes represent 4-bit zero difference; black boxes represent 4-bit non-zero differences; and boxes containing question mark represent 4-bit either zero or non-zero differences.